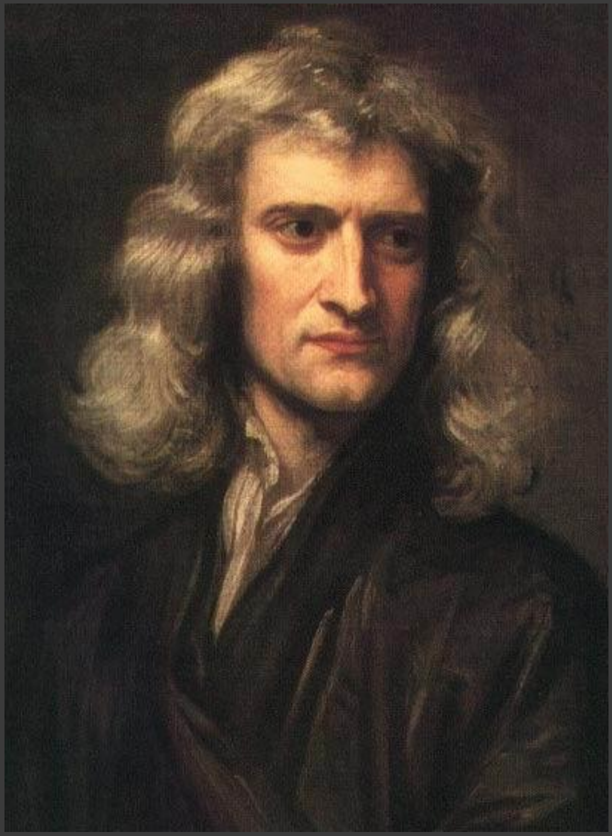
KINGS CROSS TO HONG KONG IN 42 MINUTES?

By Stephen Lucas

The idea?

Robert Hooke

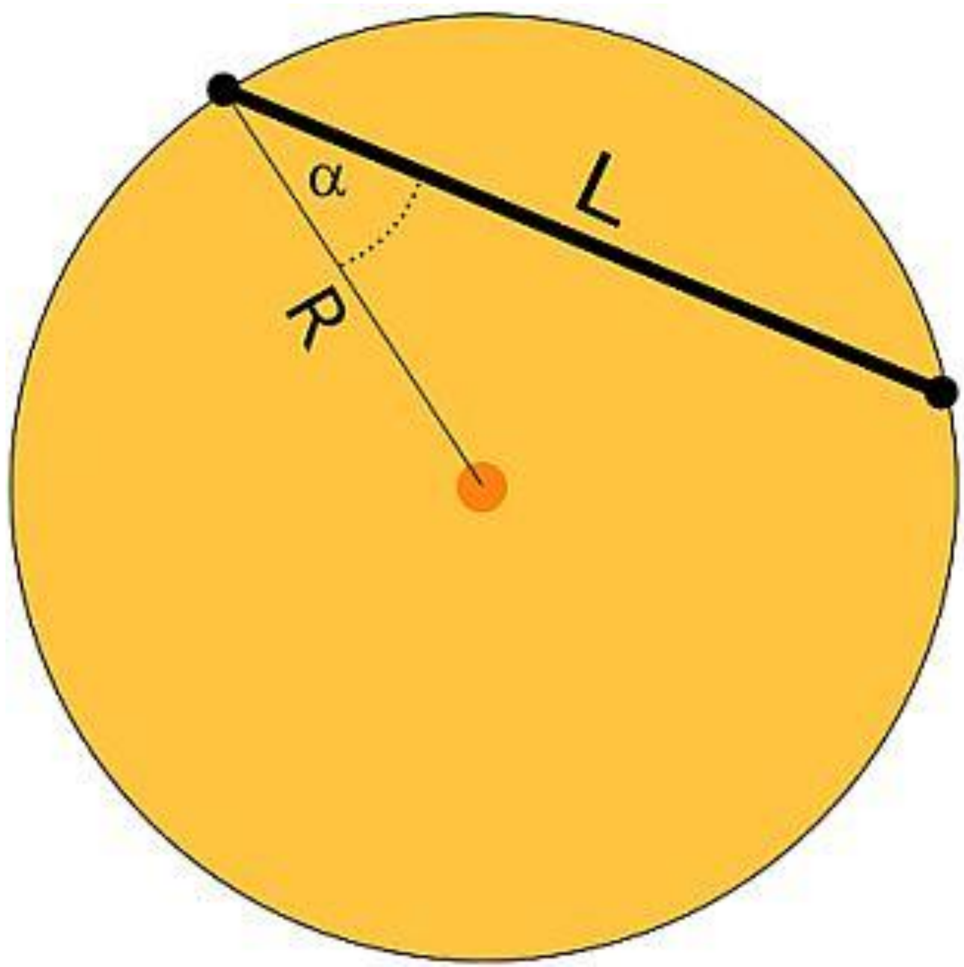


Sir Isaac Newton

Assumptions

- The Earth is a perfect sphere of uniform density.
- Air resistance and friction are negligible.
- The force due to gravity inside a homogeneous sphere is zero.
- Such materials exist that can withstand and penetrate to the core.

Mass of the Earth above the train has no influence on train's motion. The part of the Earth which is *below* the train attracts the train to the centre of the Earth with the force proportional to the volume of this part.



Draw ☺

Derivation of journey times:

To the whiteboard...

Coordinate along tunnel:

$$x = R\cos\alpha$$

M(r), mass of the Earth below the train:

$$M(r) = \frac{4\pi R^3 \rho}{3}$$

$$F = -\frac{4\pi G\rho m R}{3}$$

Only component of force acting along the tunnel affects motion of train:

$$F = mx'' = F\cos\alpha$$

$$F = \frac{-4\pi G\rho m R\cos\alpha}{3}$$

$$F = \frac{-4\pi G\rho m x}{3}$$

$$x'' = -\omega^2 x$$

Journey Time

$$\omega^2 = \frac{4\pi G\rho}{3} = \frac{g}{R}$$

$g = 9.81ms^{-2}$
$R = 6.4 \text{ x}10^6 \text{ m}$

$$\frac{T}{2} = 2537.5s = 42.3 \text{ minutes}$$

Notice how the journey time does not depend on the separation of the two points, it would take 42 minutes to go from Kings Cross to Kathmandu, as well as from Kings Cross to Kingsbury!

But will you spew sick everywhere?

- By integrating the force equation it can be shown that the potential energy is:

$$\mathrm{P.E} = \frac{2\pi G\rho m R^2}{3}$$

It then follows from the conservation of energy that:

$$\nu_{max}{}^2 = \frac{4\pi G\rho R^2}{3} = gR$$

$\nu_{max} = 7923.6\,\mathrm{ms}^{-1}$ (nyyyeeeooow!)

Will you experience more gs than a night out in Brixton?

- No!

$x'' = -\omega^2 x$

- If acceleration is maximum at maximum displacement such that x = R, we get:

$$x'' = -\omega^2 x = \frac{4\pi G\rho R}{3} = \frac{g R}{R} = g$$

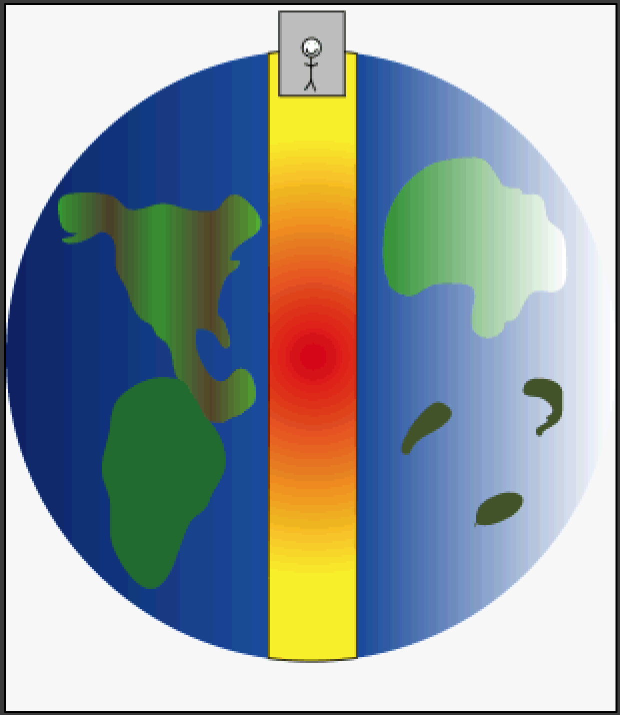
The maximum g-force experienced would be 1g, which could possibly result in 'weightlessness' depending on the direction of the train. The human body is capable of withstanding up to 4gs before blacking out therefore allowing plenty of room for the driver to do skids!

Summary

- Time taken for half the period: **42.3minutes**
- Maximum possible velocity (i.e. If going from north to south pole): $7923.6ms^{-1}$
- Maximum g-force experienced by passengers: 1g
- Average velocity: $4890.2ms^{-1}$

Pros ☺

- Would require no energy input to operate, and could operate for infinity if all frictional forces were removed completey. Even if energy input was needed, it would not be as much as other means of transportation.



- The train would naturally be coming to a stop as and when it reached the destination.
- Long haul flights are the most destructive to the environment and this method would be far quicker and greener.

Just a pipe dream? ☹

- The upheaval of such large quantities of rock and storage of this rock would be costly.
- The Earth's plate tectonics, high temperatures and pressures at the innermost core, and the fact that this core is liquid and would vaporise most materials it came into contact with.
- The friction experienced by the train would be significant, and measures to reduce friction would compromise its energy saving expenditure.
- The extremely high velocities that the passengers would experience would not give a pleasurable riding experience.
- The track alone would cost more than the GDP of the USA to construct.

Thank you for watching

...Any Questions?

Why the mass above the train has no affect on its motion:

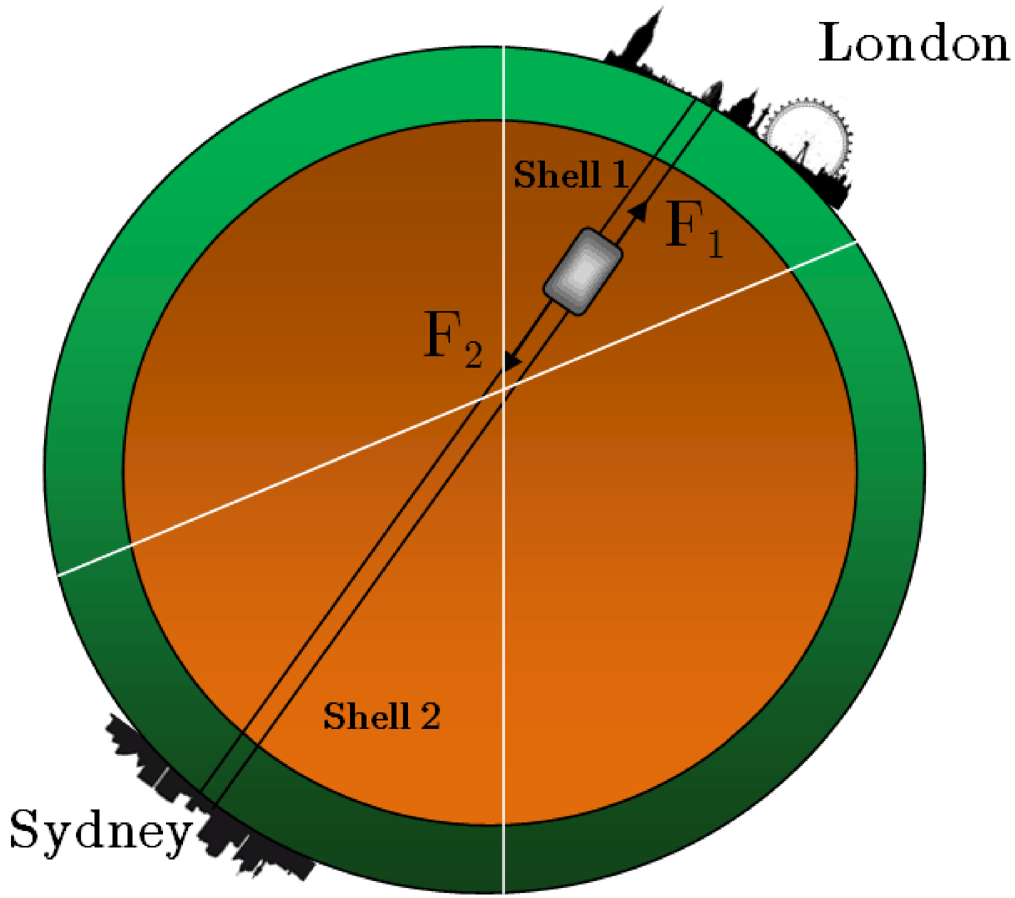


Figure 1 – Hypothetical diagram behind Hooke's idea

Newton's law of gravitation:

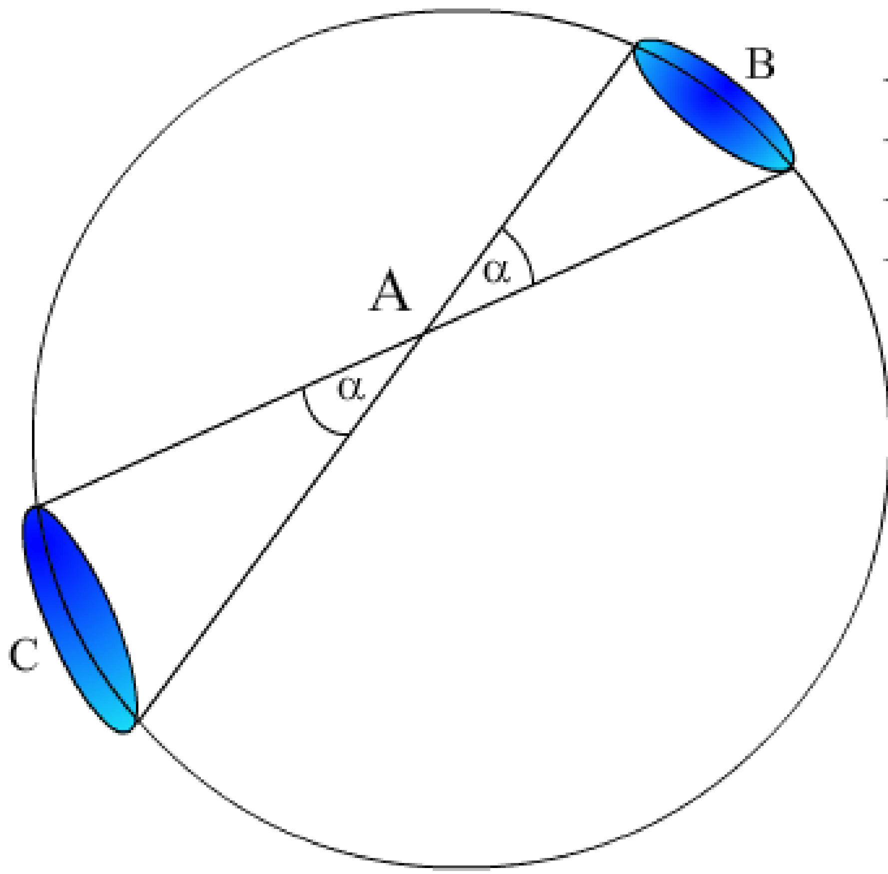
$$F = \frac{GMm}{R^2}$$

From Newton's law:

$$F_1 = F_2$$

Why?

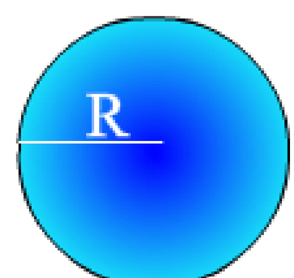
Consider extremely thin concentric sphere of uniform density:



$$M = \rho V$$

$$M \alpha \rho A_{cross}$$

$$\alpha << 1$$



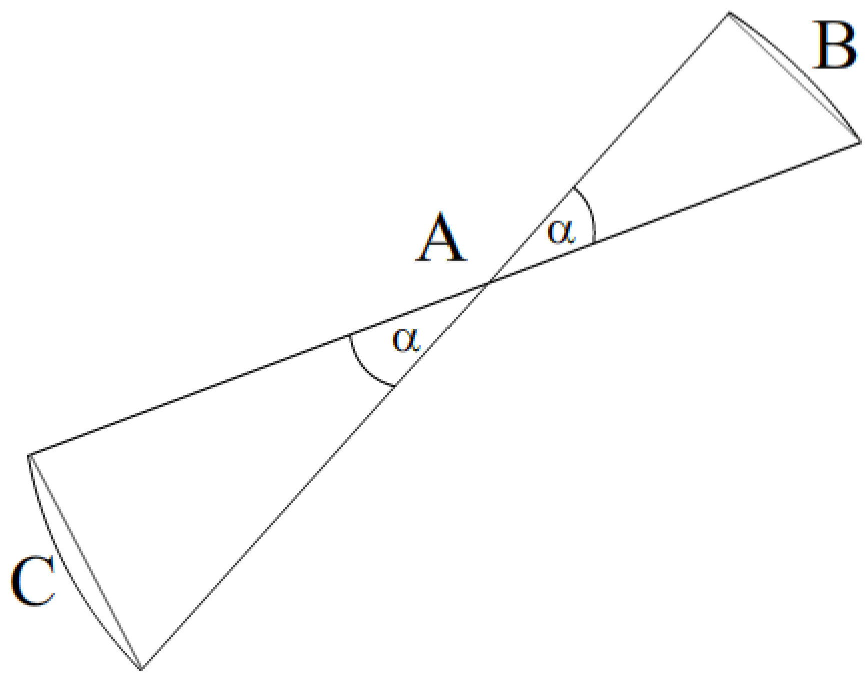
Consider mass within shells B and C:

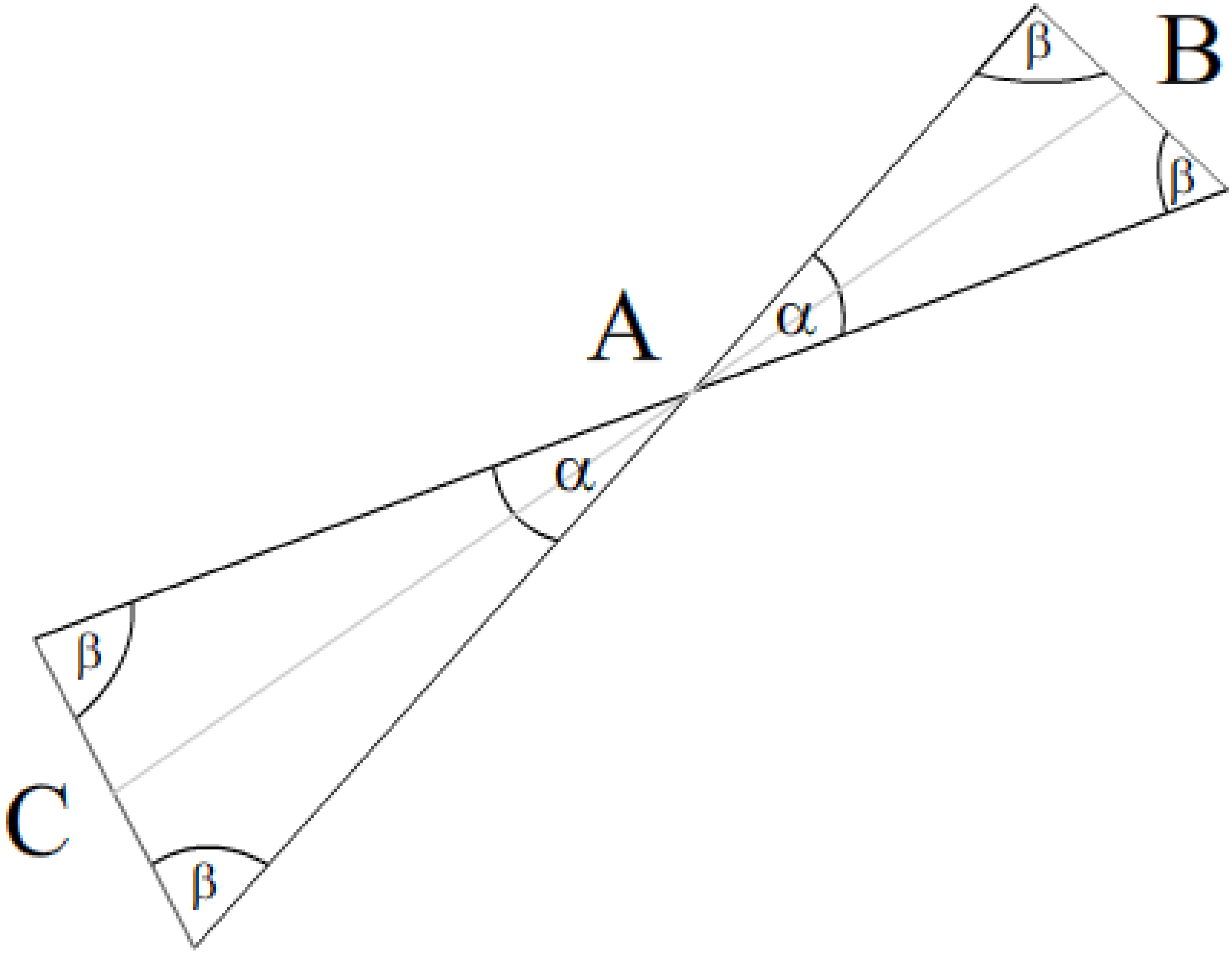
$M_B = \rho V_B = \rho A_B t$

$M_c = \rho V_c = \rho\, A_c t$

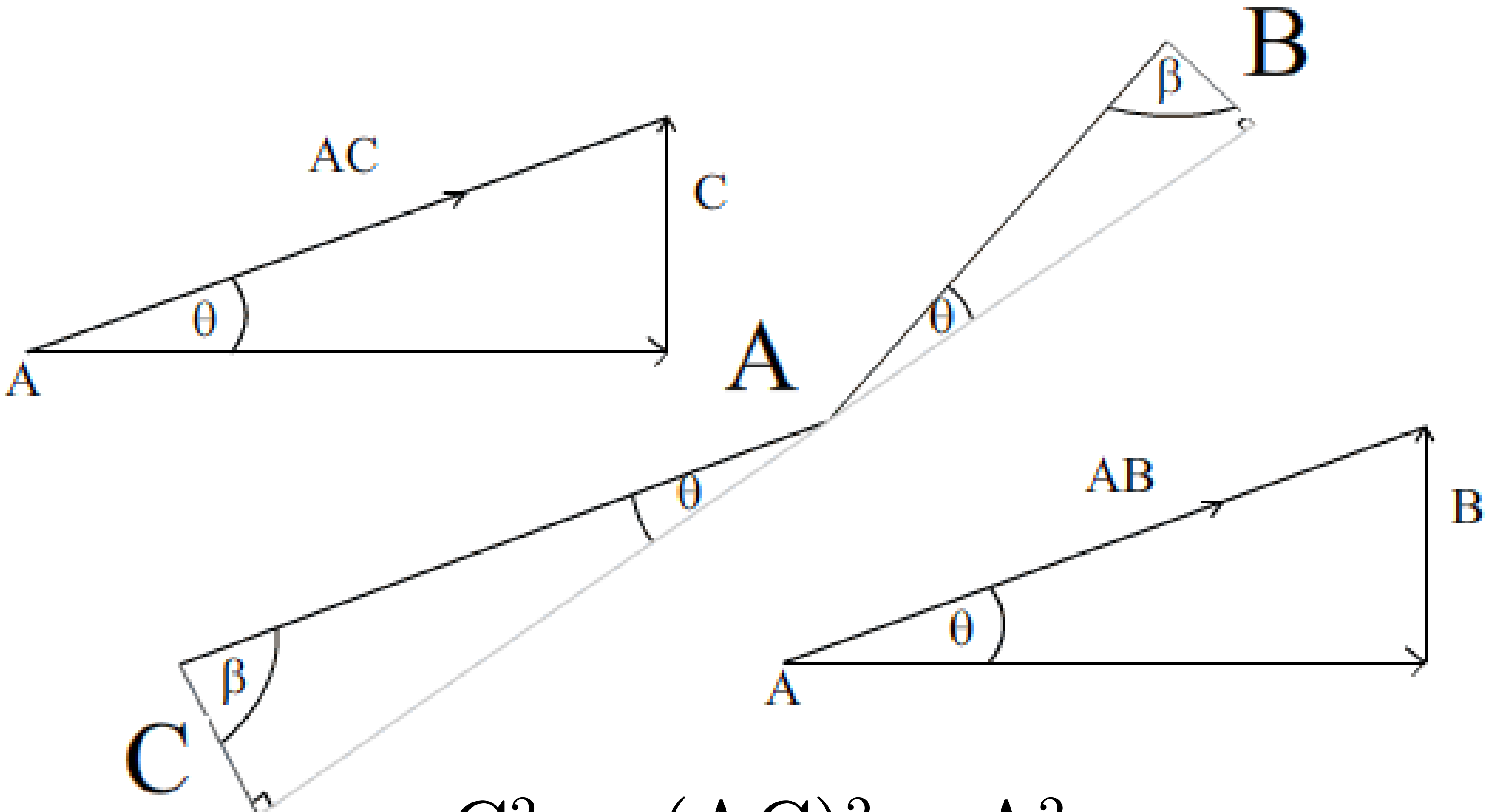
The cross-sectional area is proportional to: $(\text{radius})^2$

Radius is proportional to arc lengths of B and C.





B
β
β
A
α
α
β
C
β



$$C^2 = (AC)^2 - A^2$$
$$B^2 = (AB)^2 - A^2$$

$M_B = \rho V_B = \rho A_B t$

$M_C = \rho V_c = \rho\, A_c t$

- $M_B \propto [(AB)^2 - A^2$
- $M_C \propto (AC)^2 - A^2$
- $F_B \propto \dfrac{G^*[(AB)^2 - A^2)]m}{[(AB)^2 - A^2)]} = 4\pi t\rho_{Earth}$
- $F_C \propto \dfrac{G^*[(AC)^2 - A^2)]m}{[(AC)^2 - A^2)]} = 4\pi t\rho_{Earth}$

The forces are the same!